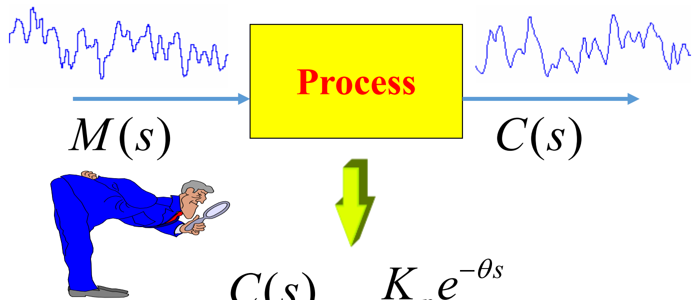
Empirical Model Identification



$$\frac{C(s)}{M(s)} = \frac{K_p e^{-\theta s}}{\tau s + 1}$$

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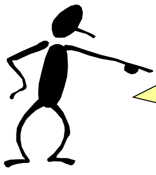
Outline

- **Experimental design for model building**
- **Process reaction curve (graphical)**
- **Statistical parameter estimation**

EMPIRICAL MODELLING



We have invested a lot of effort to learn fundamental modelling. Why are we now learning about an empirical approach?

TRUE/FALSE QUESTIONS

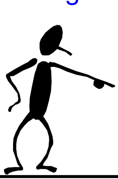
 We have all data needed to develop a fundamental model of a complex process

 We have the time to develop a fundamental model of a complex process

 Experiments are easy to perform in a chemical process

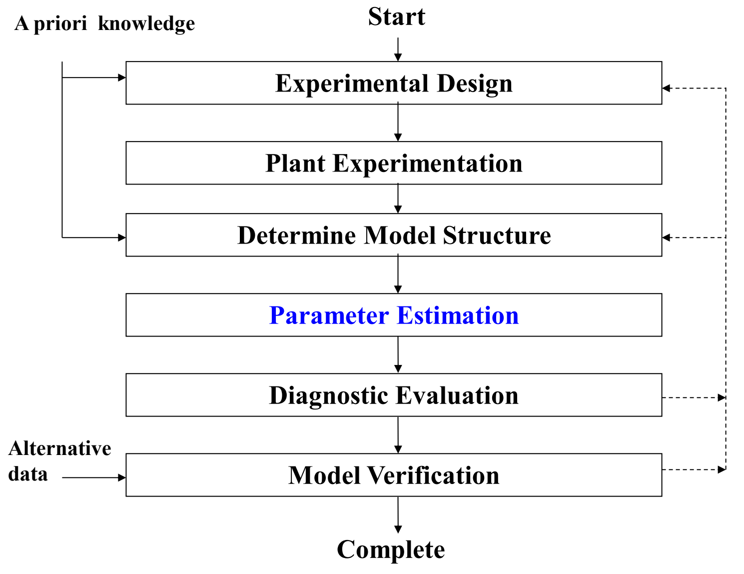
 We need very accurate models for control engineering

Fundamental Model vs.EMPIRICAL MODELLING

	Fundamental Model	Empirical (or Data-Driven) Model (AI Models)
Method	Fundamental Principles	Plant Data
Advantages	Excellent relationships between parameters in physical systems and the transient behavior of the systems	Good for process design since it is easy to use (Easy, less effort)
Disadvantages 	• Complex. ex. distillation column, 10 compounds, 50 trays 500 diff. Eqs • Large engineering effort	• Less accuracy • Do not provide enough information to satisfy all process design and analysis requirement

EMPIRICAL MODEL BUILDING PROCEDURE

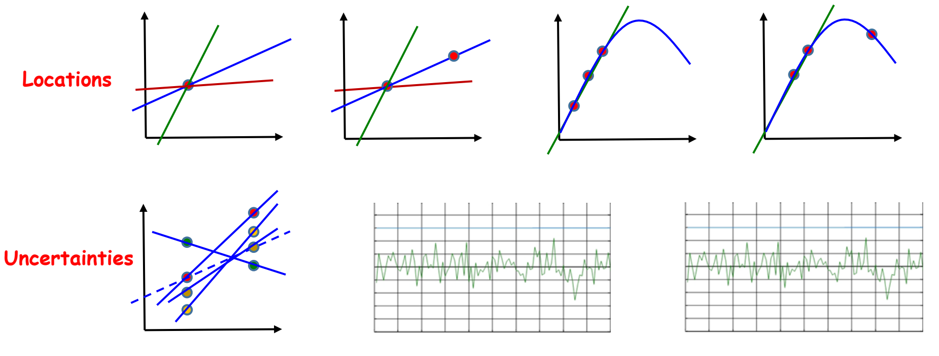
5



5

Empirical Model Identification Beyond Mere Modeling Skills

6

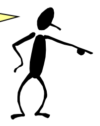


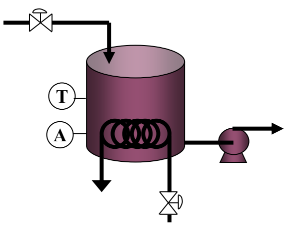
A good model relies on the quality of collected data and its ability to mitigate uncertainties.

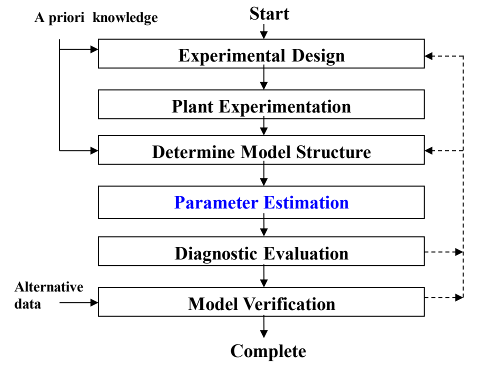
6

EMPIRICAL MODEL BUILDING PROCEDURE

Looks very general; it is! However, we still need to understand the process!

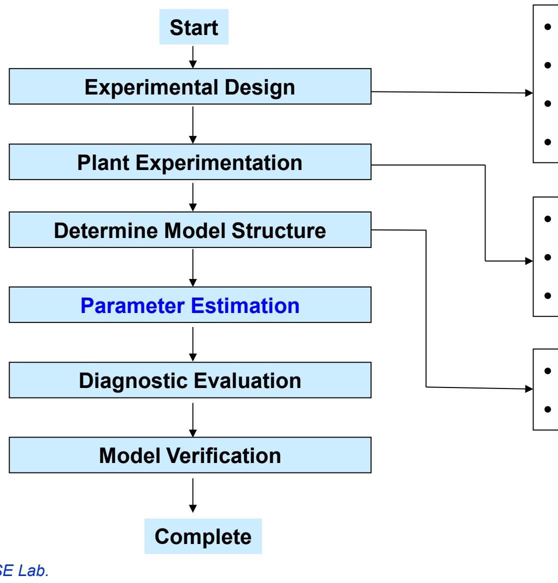






- Changing the temperature 10 K in a ethane pyrolysis reactor is allowed.
- Changing the temperature in a ?? Reactor would kill the micro-organisms

EMPIRICAL MODEL BUILDING PROCEDURE



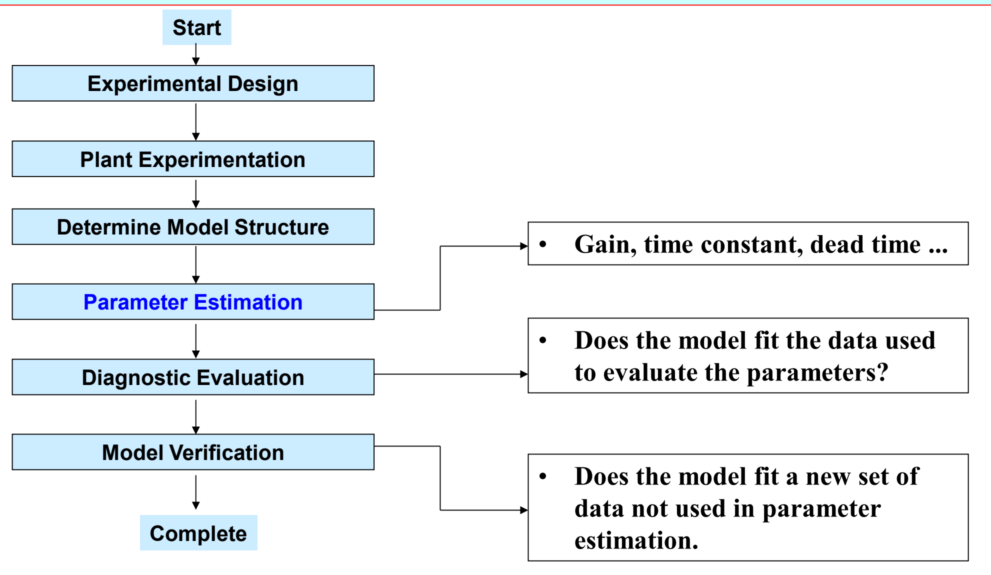
- Base case operating conditions
- Definition of perturbation
- Measures
- Duration

- Safely
- Small effect on product quality
- Small effect of profit

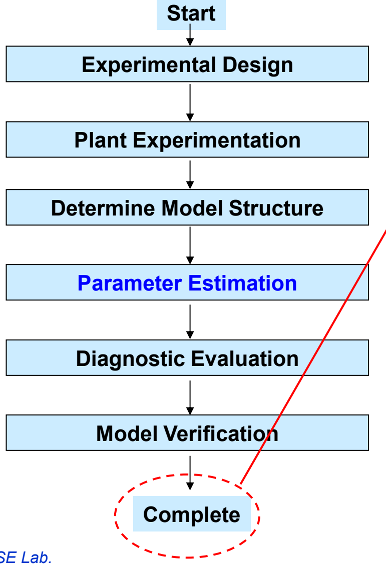
- We will stick with linear.
- What order, dead time, etc?

$$\frac{C(s)}{M(s)} = \frac{K_p e^{-\theta s}}{\tau s + 1}$$

EMPIRICAL MODEL BUILDING PROCEDURE



EMPIRICAL MODEL BUILDING PROCEDURE



- **What our goal?**

  We seek models good enough for control design, controller tuning, and process design.

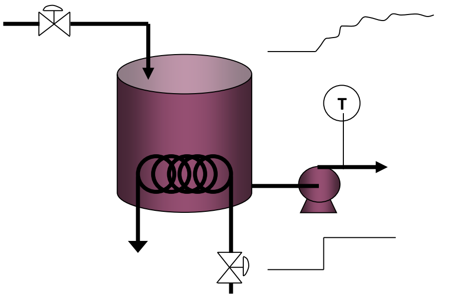
- **How do we know?**

EMPIRICAL MODEL BUILDING PROCEDURE

11

Process reaction curve - The simplest and most often used method. Gives nice visual interpretation as well.

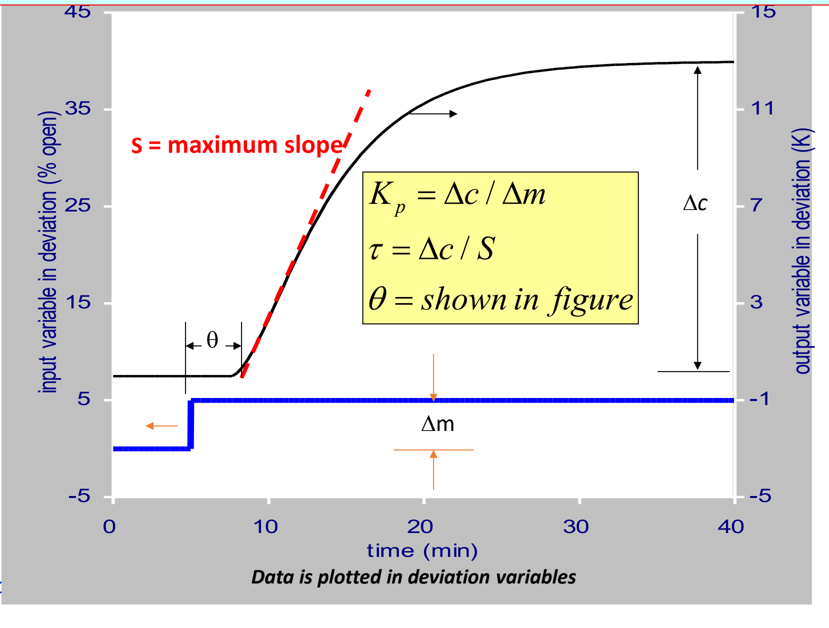
1. Start at steady state
2. Single step to input
3. Collect data until steady state
4. Perform calculations



11

Method (fit 1): Process Reaction Curve

12



Input change: Δm

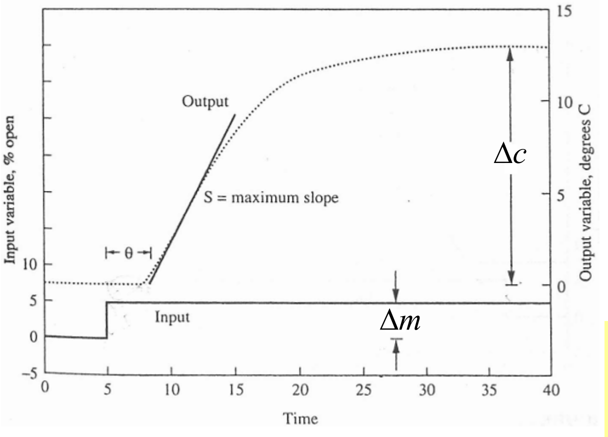
Output change: ΔC

$$\frac{C(s)}{M(s)} = \frac{K_p e^{-\theta s}}{\tau s + 1}$$

12

13

Method: Process Reaction Curve



Model: $G(s) = \dfrac{2.6}{9.36s+1} e^{-3.3s}$

$\Delta m = 5\% \quad open \qquad \Delta c = 13.1^\circ C$

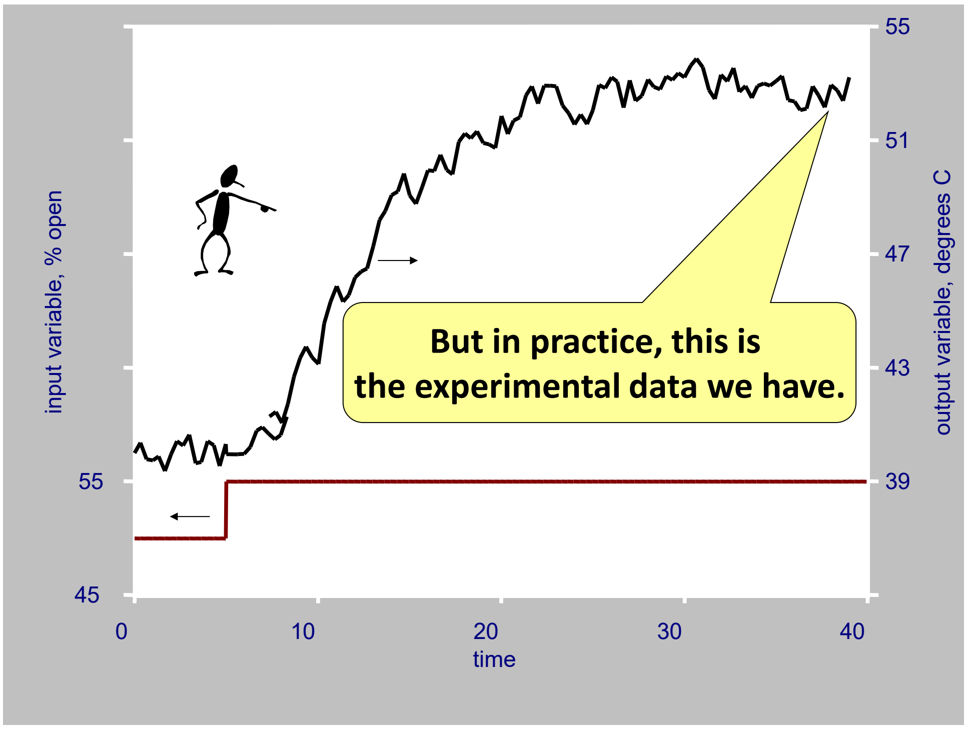
$K_p = \Delta c / \Delta m = 13.1^\circ C/(5\%\, open) = 2.6^\circ C/\%\, open$

$s = 1.4^\circ C/\min$

$\tau = \Delta c / s = 13.1^\circ C/(1.40^\circ C/\min) = 9.36\,\min$

13

14



14

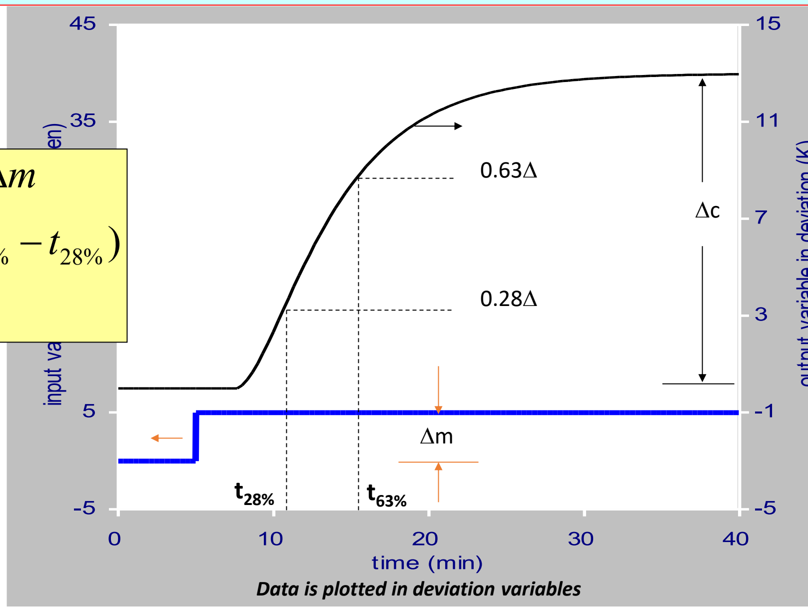
15

Method (fit 3): Process Reaction Curve

$$K_p = \Delta c / \Delta m$$

$$\tau = 1.5\ (t_{63\%} - t_{28\%})$$

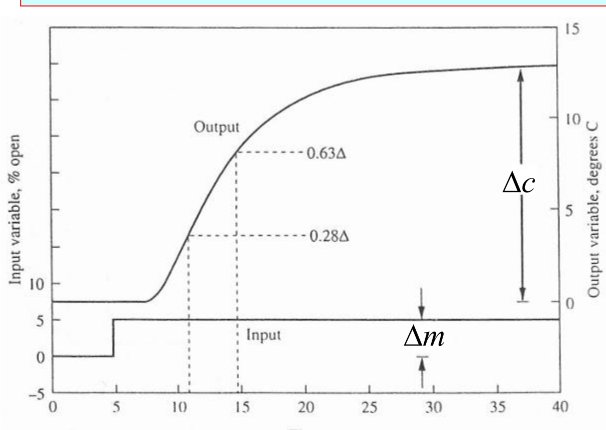
$$\theta = t_{63\%} - \tau$$



15

16

Method (fit 3): Process Reaction Curve



Model: $$G(s) = \frac{2.6}{6s+1} e^{-3.7s}$$

$$0.63\Delta c = 8.3^\circ C \quad , \quad t_{63\%} = 9.7 \text{ min}$$

$$0.28\Delta c = 3.7^\circ C \quad , \quad t = 5.7 \text{ min}$$

$$\tau = 1.5(t_{63\%} - t_{28\%}) = 1.5(9.7 - 5.7) = 6 \text{ min}$$

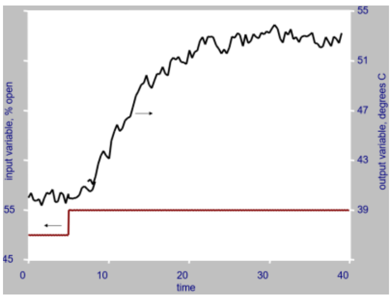
$$\theta = t_{63\%} - \tau = 9.7 - 6 = 3.7 \text{ min}$$

$$K_p = \Delta c / \Delta m = 13.1/5 = 2.6$$

16

17

Process Reaction Curve



fit I

- Developed first
- Prone to errors because of evaluation of maximum slope

fit III

- Developed in 1960's
- Simple calculations

Recommended

NOTE: Because of the difficulty in evaluating the slope, especially when the signal has high frequency noise, fit I has larger error in the parameter estimates; thus fit III is preferred.

17

18

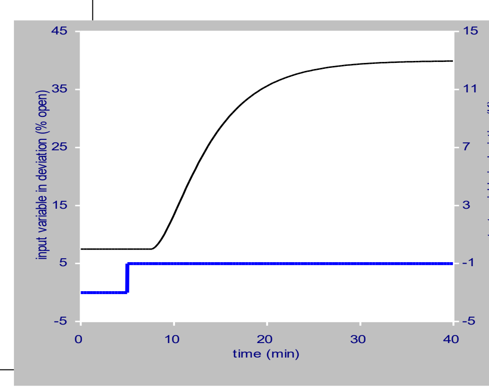
Statistical Method

Provides much more general approach that is not restricted to

- step input
- first order with dead time model
- single experiment
- "large" perturbation
- attaining steady-state at end of experiment

Requires

- more complex calculations



18

19

Statistical Method

- **The basic idea is to formulate the model we want so that regression can be used to evaluate the parameters.**
- **Now we still do this for a first order plus dead time model, although the method is much more general.**

$$\tau \frac{dY(t)}{dt} + Y(t) = K_p U(t-\theta) \qquad \frac{Y(s)}{U(s)} = \frac{K_p e^{-\theta s}}{\tau s + 1}$$

20

Statistical Method

We have discrete measurements, so let's express the model as a difference equation, with the next prediction based on current and past measurements.

$$G(s) = \frac{K_P}{\tau s + 1} e^{-\theta s} \quad \Leftrightarrow \quad \tau \frac{dY(t)}{dt} + Y(t) = K_p U(t-\theta)$$

$$\tau \frac{dy(t)}{dt} + y(t) = K_p u(t-\theta) \qquad y(t=0) = y(0)$$

Not at steady-state condition

$$y_{k+1} = a y_k + b u_{k-\Gamma}, \quad \begin{cases} a = e^{-\Delta t / \tau} \\ b = K_p (1 - e^{-\Delta t / \tau}) \\ \Gamma = \theta / \Delta t \end{cases}$$

Statistical Method

21

$$\tau \frac{dy(t)}{dt} + y(t) = K_p x(t)$$

Assume that the input is constant at the value of x_k **over** $\left[t_{k-1} \quad t_k\right]$

$$\tau s Y(s) - \tau y_{k-1} = K_p X(s) - Y(s) \Rightarrow Y(s) = \frac{\tau y_{k-1}}{\tau s + 1} + \frac{K_p X(s)}{\tau s + 1}$$

$$y_k = e^{-\Delta t/\tau} y_{k-1} + K_p (1 - e^{-\Delta t/\tau}) x_{k-1}$$

If dead time exists:

$$\Gamma = \text{int}(\frac{\theta}{\Delta t}) \Rightarrow y_k = e^{-\Delta t/\tau} y_{k-1} + K_p (1 - e^{-\Delta t/\tau}) x_{k-\Gamma-1}$$

$$a = e^{-\Delta t/\tau}$$
$$b = K_p (1 - e^{-\Delta t/\tau})$$
$$\Gamma = \theta / \Delta t$$

21

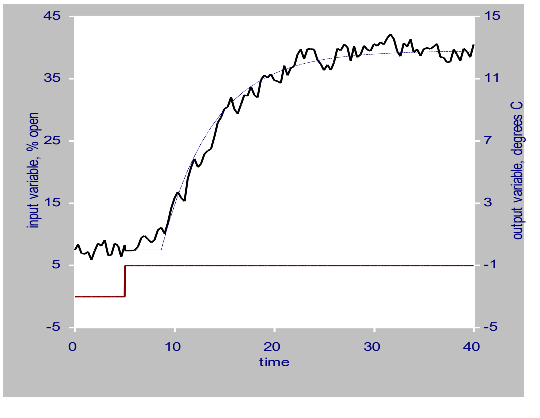
Statistical Method I

22

With discrete measurements, let's express the model as a difference equation. It is used for the next prediction based on current and past measurements.

$$\left(y_{k+1}\right)_{predicted} = a\left(y_k\right)_{measured} + b\left(u_{k-\Gamma}\right)_{measured}$$

$$\min \sum_k \left[\left(y_{k+1}\right)_{measured} - \left(y_{k+1}\right)_{predicted}\right]^2$$



22

Statistical Method I

23

$$\min_{a,b} \sum_{k=\Gamma+1}^{K} E_{k+1}^2 = \min_{a,b} \sum_{k=\Gamma+1}^{K} \left[(y_{k+1})_m - (y_{k+1})_p \right]^2$$

$(y_k)_m$: measured system output

$(y_k)_p$: predicted from process model

$$= \min_{a,b} \sum_{k=\Gamma+1}^{K} \left[(y_{k+1})_p - \left\{ a(y_k)_m + b(u_{k-\Gamma})_m \right\} \right]^2$$

$$\frac{\partial}{\partial a}\left[\sum_{k=\Gamma+1}^{K} E_k^2\right] = -2\sum_{k=\Gamma+1}^{K} (y_k)_m \left[(y_{k+1})_m - a(y_k)_m - b(u_{k-\Gamma})_m\right] = 0$$

$$\frac{\partial}{\partial b}\left[\sum_{k=\Gamma+1}^{K} E_k^2\right] = -2\sum_{k=\Gamma+1}^{K} (u_{k-\Gamma})_m \left[(y_{k+1})_m - a(y_k)_m - b(u_{k-\Gamma})_m\right] = 0$$

$$a\sum_{k=\Gamma+1}^{K} (y_k)_m^2 + b\sum_{k=\Gamma+1}^{K} (y_k)_m (u_{k-\Gamma})_m = \sum_{k=\Gamma+1}^{K} (y_k)_m (y_{k+1})_m$$

$$a\sum_{k=\Gamma+1}^{K} (y_k)_m (u_{k-\Gamma}) + b\sum_{k=\Gamma+1}^{K} (u_{k-\Gamma})_m^2 = \sum_{k=\Gamma+1}^{K} (u_{k-\Gamma})_m (y_{k+1})_m$$

23

Statistical Method I

24

$$a\sum_{k=\Gamma+1}^{K} (y_k)_m^2 + b\sum_{k=\Gamma+1}^{K} (y_k)_m (u_{k-\Gamma})_m = \sum_{k=\Gamma+1}^{K} (y_k)_m (y_{k+1})_m$$

$$a\sum_{k=\Gamma+1}^{K} (y_k)_m (u_{k-\Gamma}) + b\sum_{k=\Gamma+1}^{K} (u_{k-\Gamma})_m^2 = \sum_{k=\Gamma+1}^{K} (u_{k-\Gamma})_m (y_{k+1})_m$$

$$\begin{bmatrix} \sum_{k=\Gamma+1}^{K} (y_k)_m^2 & \sum_{k=\Gamma+1}^{K} (y_k)_m (u_{k-\Gamma})_m \\ \sum_{k=\Gamma+1}^{K} (y_k)_m (u_{k-\Gamma})_m & \sum_{k=\Gamma+1}^{K} (x'_{i-\Gamma})_m^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum_{k=\Gamma+1}^{K} (y_k)_m (y_{k+1})_m \\ \sum_{k=\Gamma+1}^{K} (u_{k-\Gamma})_m (y_{k+1})_m \end{bmatrix}$$

solve for a & b

24

Statistical Method II

25

$$\min_{a,b}\sum_{k=\Gamma+1}^{K}E_{k+1}^2=\min_{a,b}\sum_{k=\Gamma+1}^{K}\left[\left(y_{k+1}\right)_m-\left(y_{k+1}\right)_p\right]^2=\min_{a,b}\sum_{k=\Gamma+1}^{K}\left[\left(y_{k+1}\right)_m-\left\{a\left(y_k\right)_m+b(u_{k-\Gamma})_m\right\}\right]^2$$

⇩

$$\left(y_{k+1}\right)_p\simeq a\left(y_k\right)_m+b(u_{k-\Gamma})_m$$

$$\underbrace{\begin{bmatrix} y_3 & u_{3-\Gamma}\\ y_4 & u_{4-\Gamma}\\ \vdots & \vdots\\ y_{K-1} & u_{k-\Gamma-1}\end{bmatrix}}_{\mathbf{U}}\begin{bmatrix}a\\ b\end{bmatrix}=\underbrace{\begin{bmatrix}y_4\\ y_5\\ \vdots\\ y_K\end{bmatrix}}_{\mathbf{y}}\Leftrightarrow\mathbf{U}\begin{bmatrix}a\\ b\end{bmatrix}=\mathbf{y}$$

$$\mathbf{U}^T\mathbf{U}\begin{bmatrix}a\\ b\end{bmatrix}=\mathbf{U}^T\mathbf{y}\Leftrightarrow\begin{bmatrix}a\\ b\end{bmatrix}=\left[\mathbf{U}^T\mathbf{U}\right]^{-1}\mathbf{U}^T\mathbf{y}$$

solve for a & b

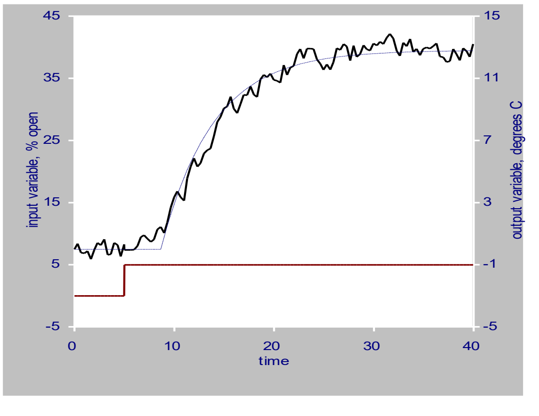
25

Statistical Method

26

Assumption for statistical method

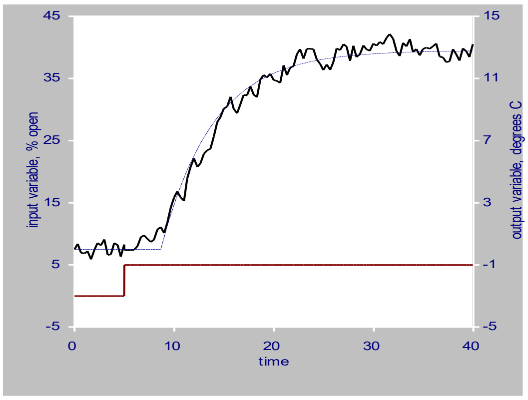
- **If Γ is unknown, solve least square problem for several Γs, find Γ with lowest E**
- **The error E is an independent random variables with zero mean**
- **The (FOPDT) model structure reasonable represents the true process dynamics**
- **The parameters *a* and *b* do not change significantly during the experiment**



26

27

Statistical Method



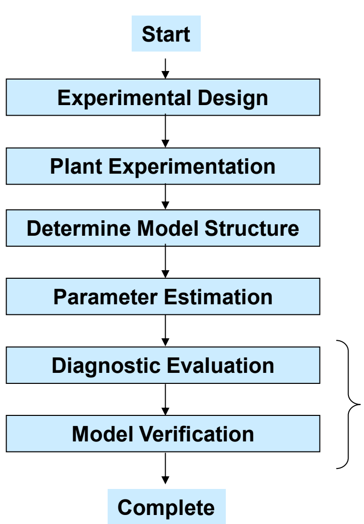
Sampling time: $\Delta t = 0.33 \text{min}$

dead time, Γ	a	b	$\sum E^2$
7	0.9640	0.1010	7.52
8	0.9605	0.1080	6.33
9	0.9578	0.1143	5.86
10	0.9555	0.1196	6.21

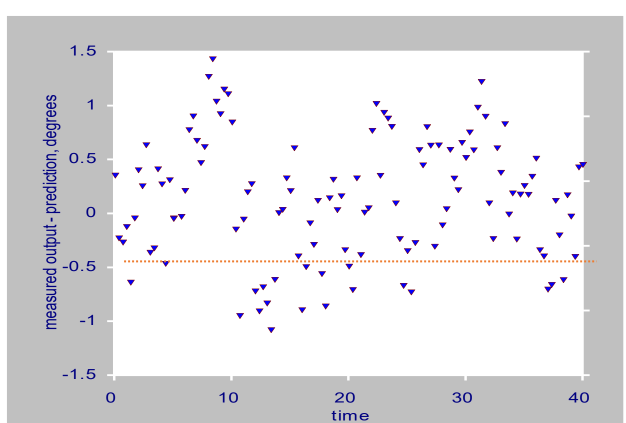
27

28

Statistical Method



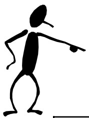
$\left[(y_k)_{predicted} - (y_k)_{measured} \right]$ **Random?**



Plotted for every measurement (sample)

28

EMPIRICAL MODEL BUILDING PROCEDURE

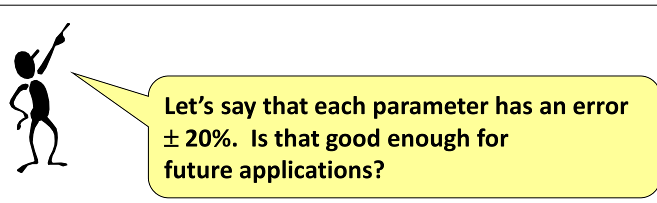


Feature	Process reaction curve	Statistical method
Input magnitude	Signal/noise > 5	Can be much smaller
Experiment duration	Reach steady state	Steady state not required
Input change	Nearly perfect step	Arbitrary, not sufficient “information” required
Model structure	First order with dead time	General linear dynamic model
Accuracy with unmeasured disturbances	Poor with significant disturbance	Poor with significant disturbance
Diagnostics	Plot prediction vs data	Plot residuals
Calculations	simple	Requires spreadsheet or other computer program

EMPIRICAL MODEL BUILDING PROCEDURE

How accurate are empirical models?

- **Linear approximations of non-linear processes**
- **Influence of noise and unmeasured disturbances on data**
- **Imperfect implementation of valve change**
- **Sensor errors**



On-Line Model Verification

31

$$\frac{K_p e^{-\theta s}}{\tau s + 1}$$

Check online whether the difference between actual plant output and model prediction is within acceptable limits.

31